Magic Adversaries *VS* Individual Reduction
--- " Science Wins Either Way "

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We show that there must be a new way to get around some of known BB lower bounds.

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- Generate a lone line of research [CLOS02, PR03, Lin03b, PR05, Pas04, Lin08, GGJ13, GGJS12, GGS15, GLP+15…];
- Known constant-round CZK protocols rely on much stronger assumption [CLP15,PPS15]

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Proof idea.

Given a magic adv V^* that breaks the dist. CZK of Feige-Shamir, we construct PKE/KE from V^* (based on injective OWF).

The classic Feige-Shamir Argument

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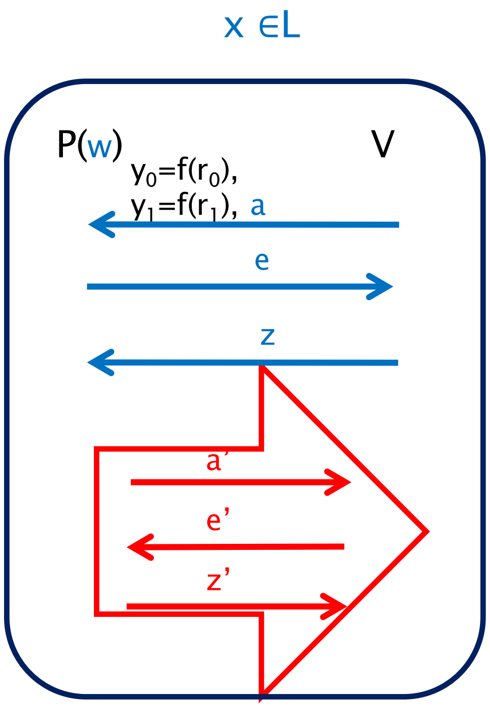
In Standalone setting



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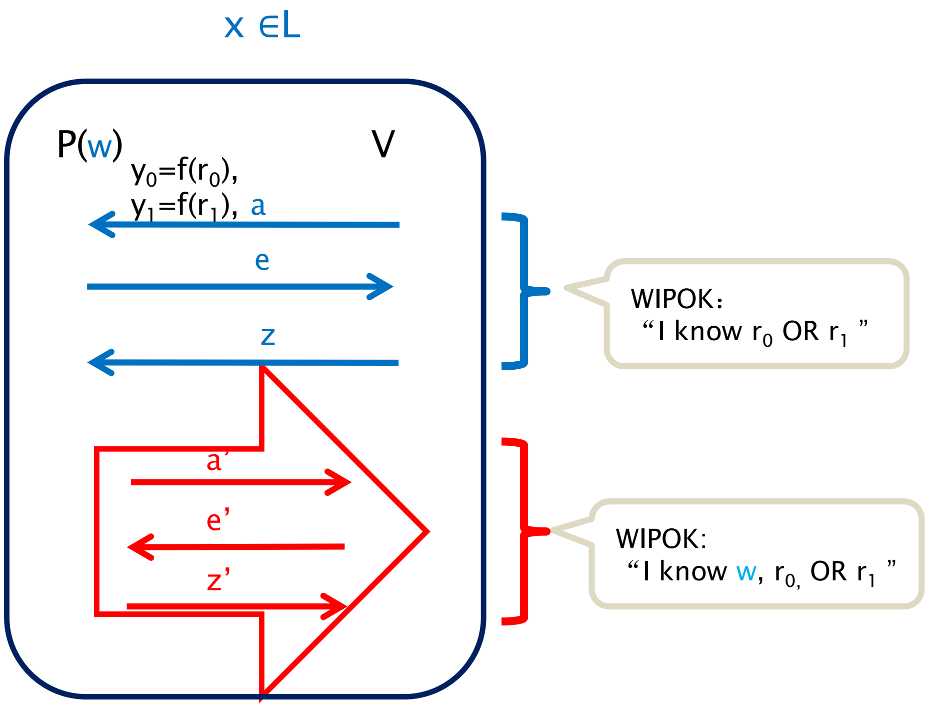

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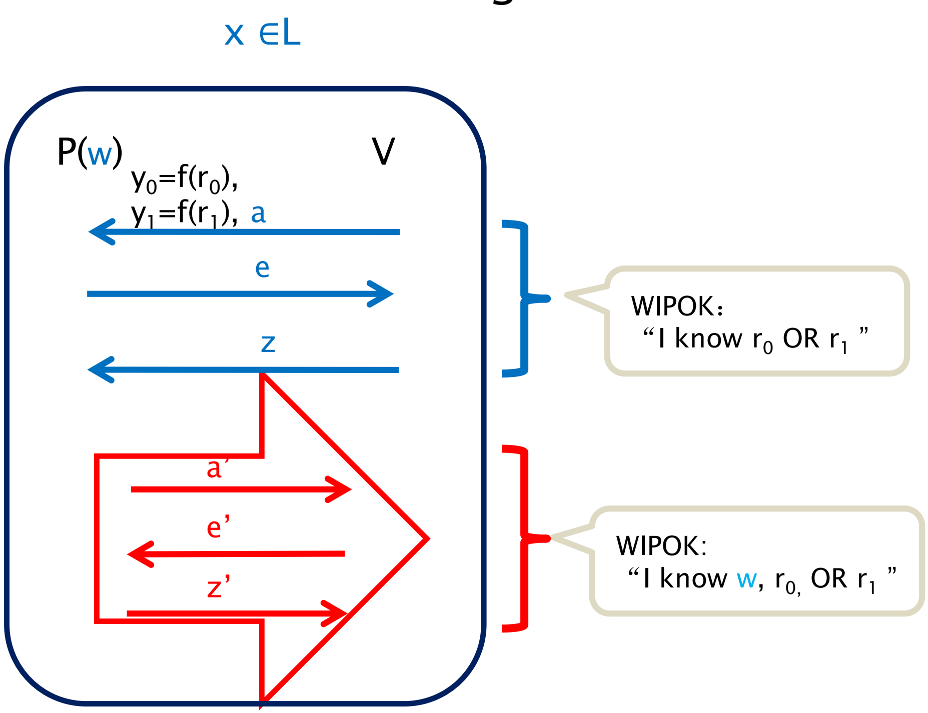

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- Soundness;

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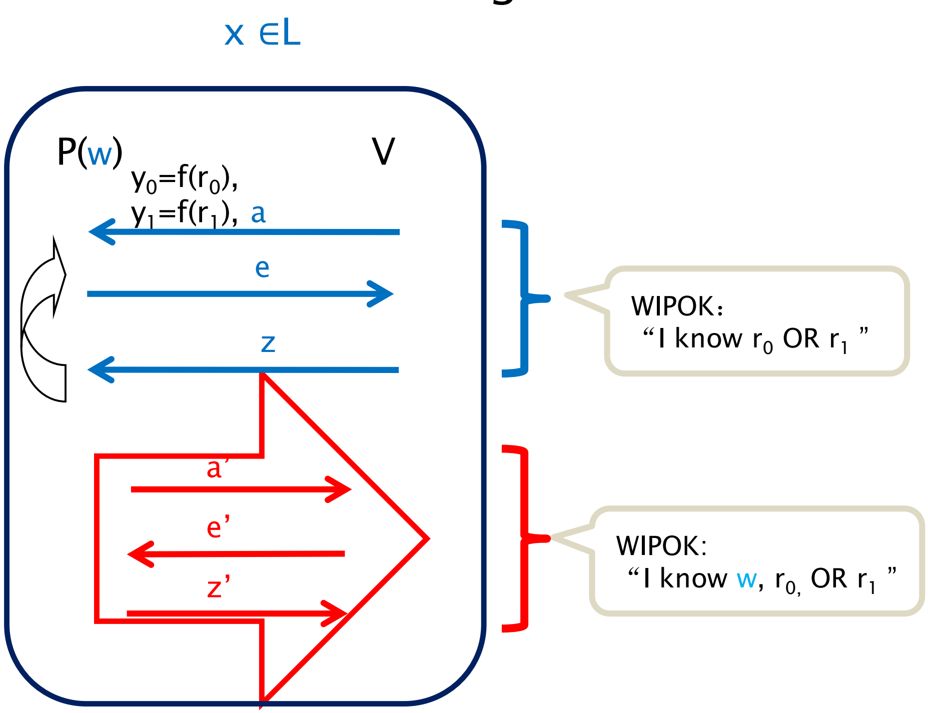

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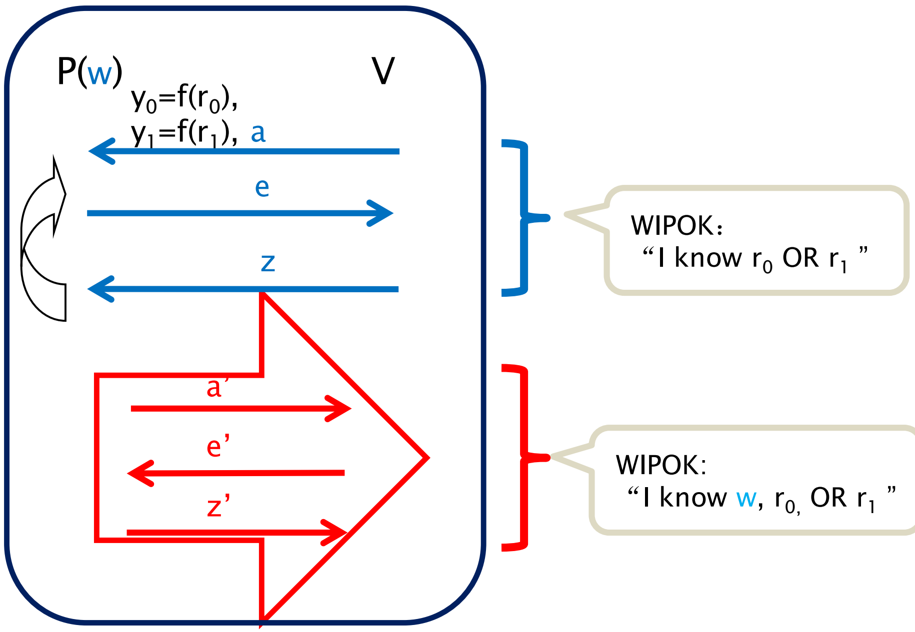
$x \in L$



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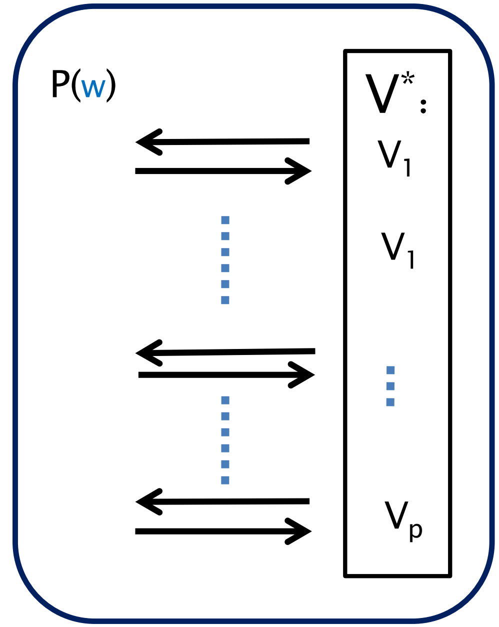
In concurrent setting



V^*controls all msgs scheduling.

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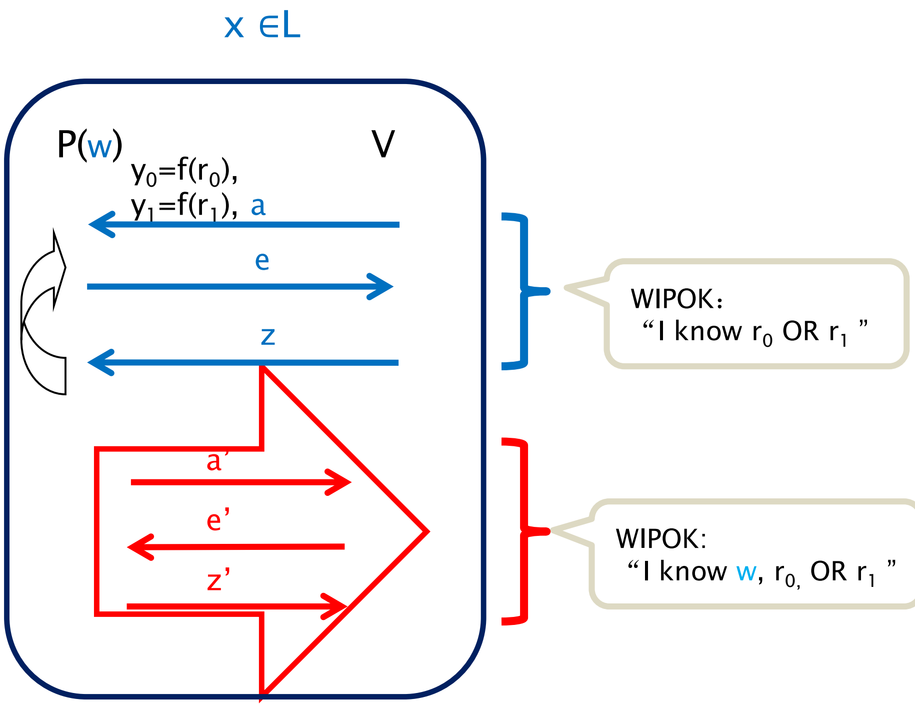

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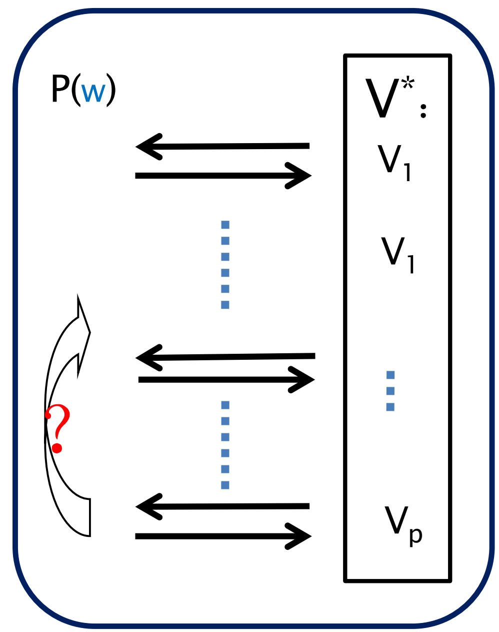
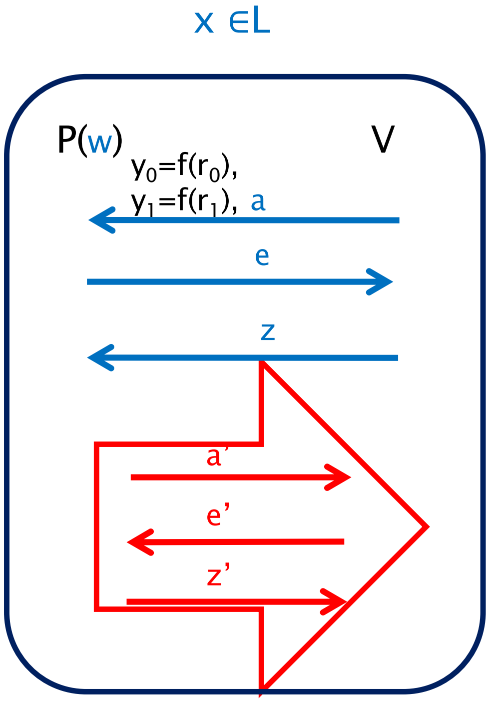


V^*controls all msgs scheduling.

BB simulator fails: Nesting effect

In fact

F-S in Standalone setting



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- For any $o(\log n/\log\log n)$-round protocol (e.g. Feige-Shamir) , there is a class C of concurrent verifiers for which BB simulator fails [CKPR01]:

$$\nexists \text{ (bb) S } \forall \ \mathcal{V} \in C$$

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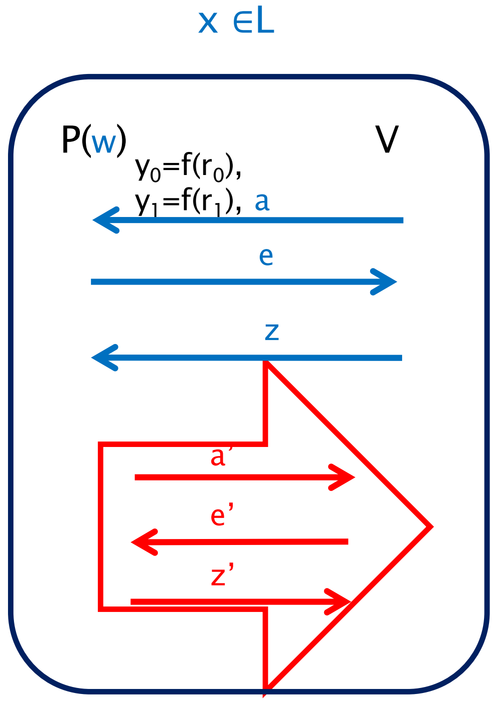

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- We observe that for every $\mathcal{V} \in \mathcal{C}$, there is a simulator that works well:

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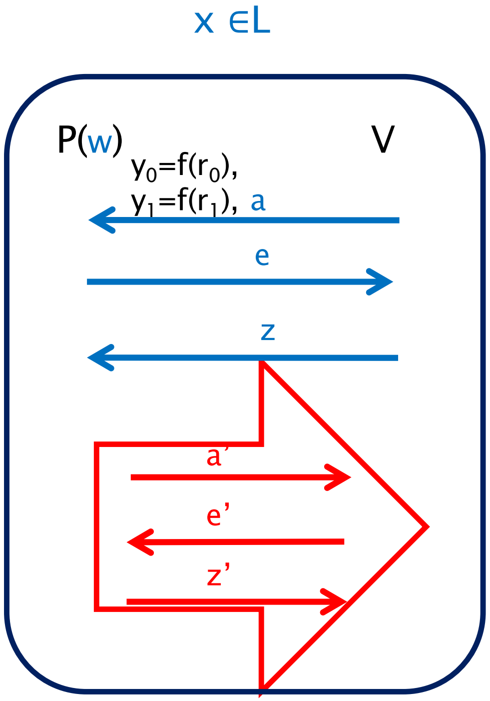

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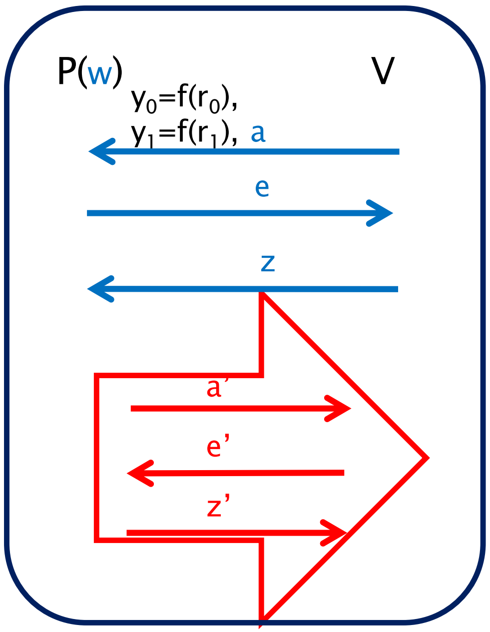
$\text{S}_{\mathcal{V}}$ takes the randomness and functionality of $\mathcal{V}$ as input.

F-S in Standalone setting

x ∈L



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Natural security definitions only require the *existence* of reduction/simulation.

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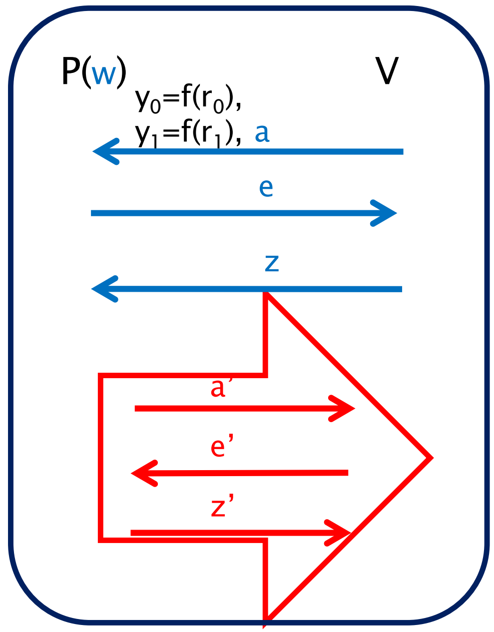

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This reveals a gap between the *universal* simulation

$$\exists \text{ S } \forall\ \mathcal{V}$$

and *individual* simulation

$$\forall\ \mathcal{V}\ \exists \text{ S}$$

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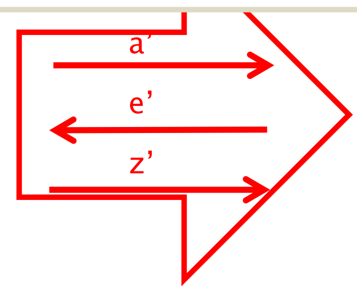

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Any magic adv V* (not in C) that breaks CZK of Feige-Shamir (i.e., no efficient alg can simulate its view) ?

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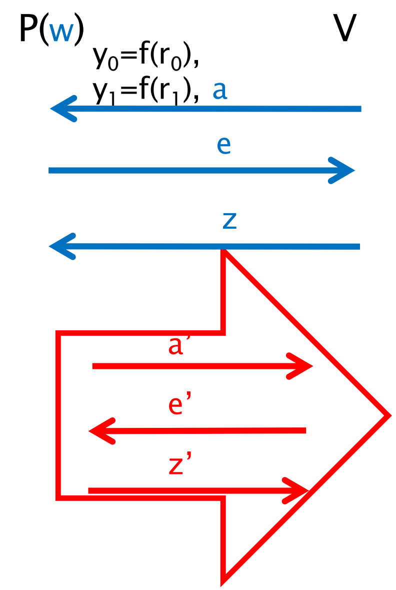

Consequence of a magic adv V^* *(oversimplified)*

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Fix n， and assume V^* runs in poly(n) steps in a real interaction.

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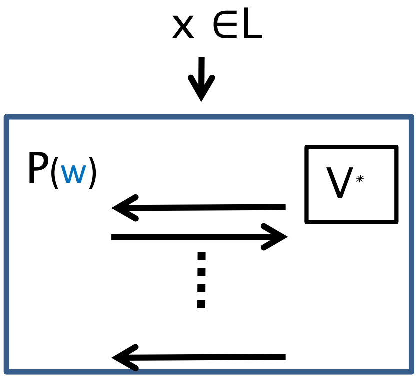
$\exists$ V^*' step i:



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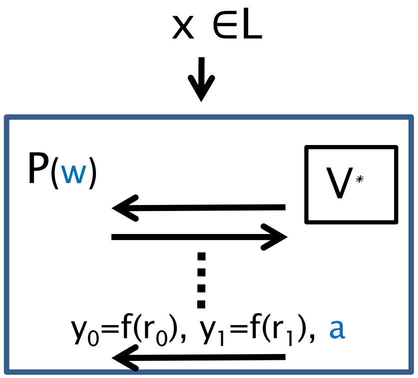

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∃ $V^{*'}$ step i:

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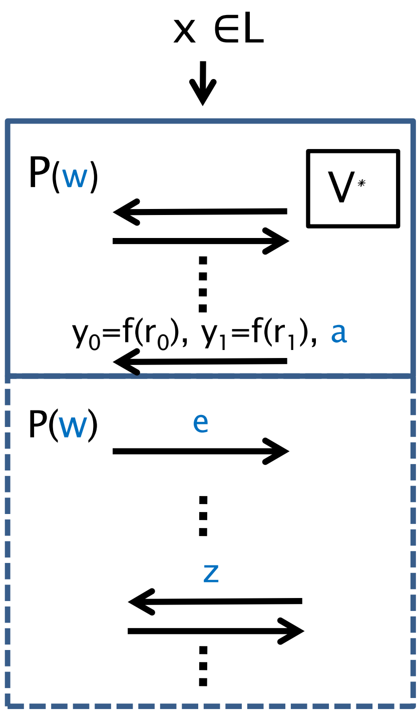

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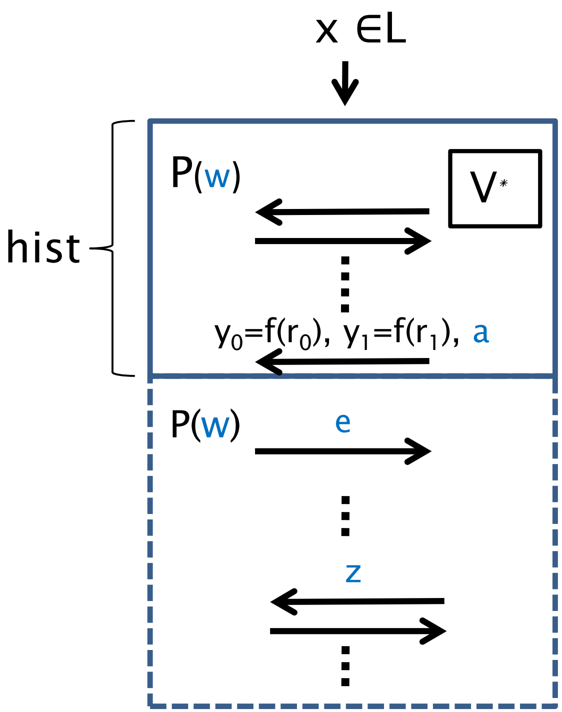

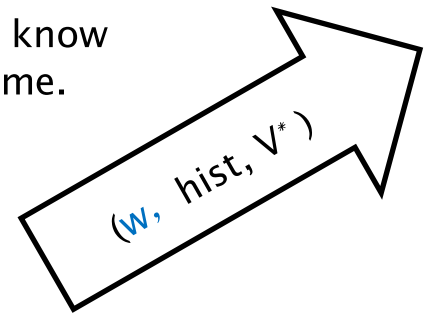

Given the witness w as input, there is a PPT inverting E,

E (w,hist, V^*) → r_b.

(by rewinding)

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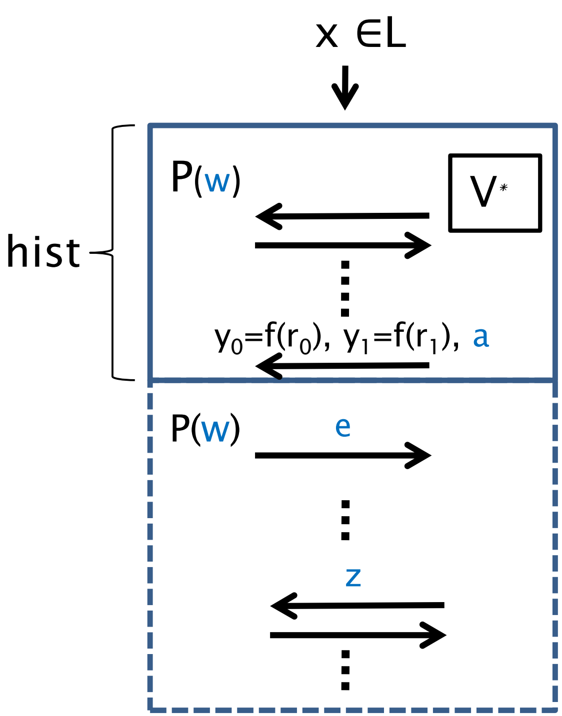

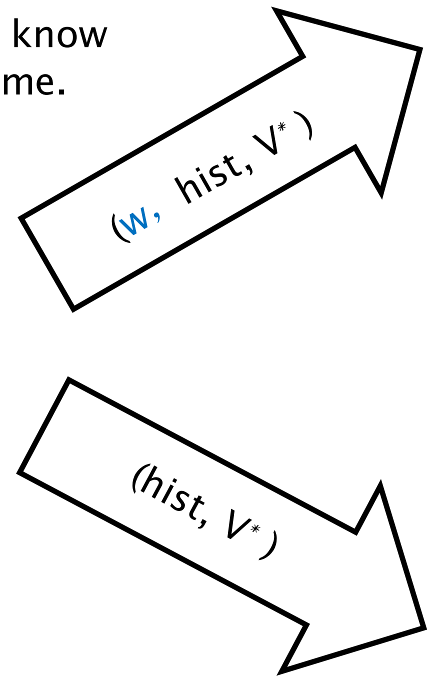

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(otherwise we will have a simulator for V^*)

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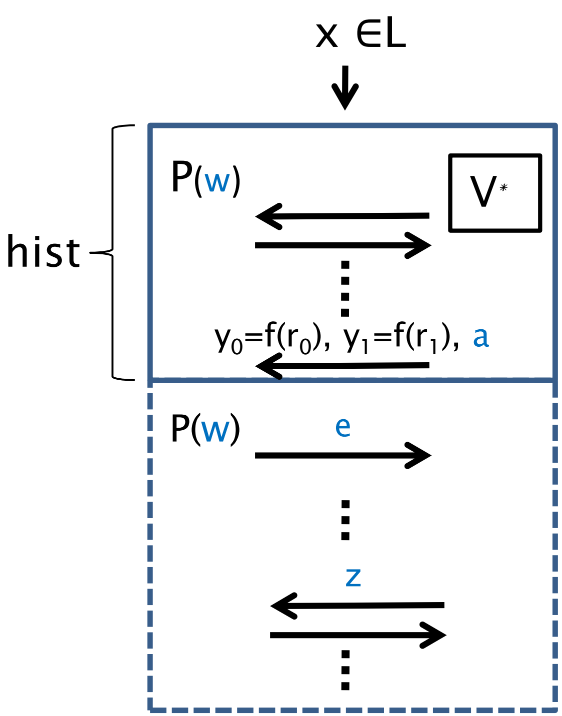

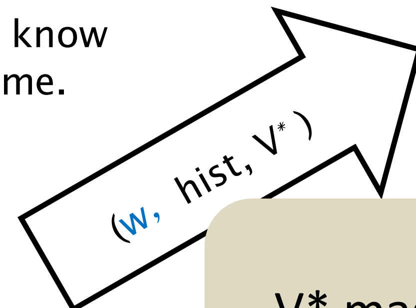

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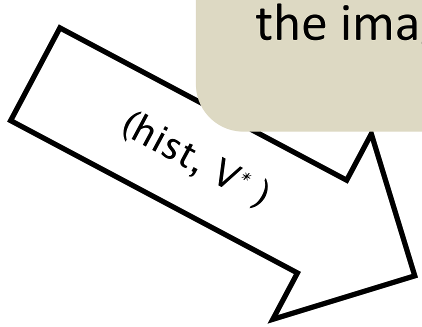
V* magically creates a trapdoor (w) for the images of f output by V^* at its step i.



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Actually, we prove that there are infinitely many n, for each n

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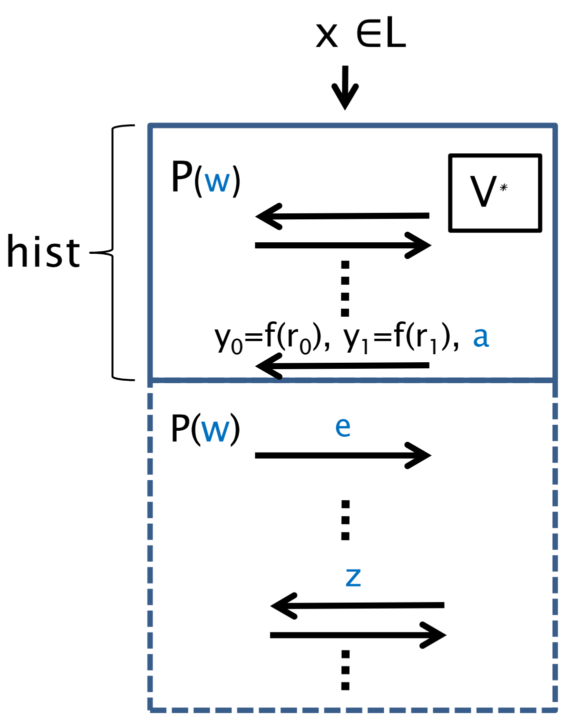

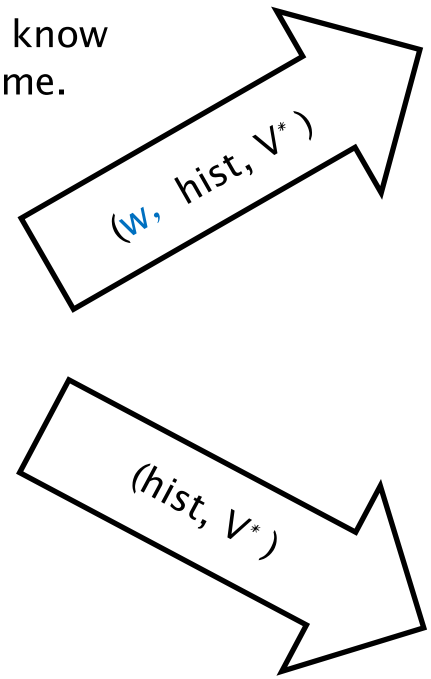

Given the witness w as input, there is a PPT extractor E, E (w,hist, V^*) can extract r_b. (by rewinding)

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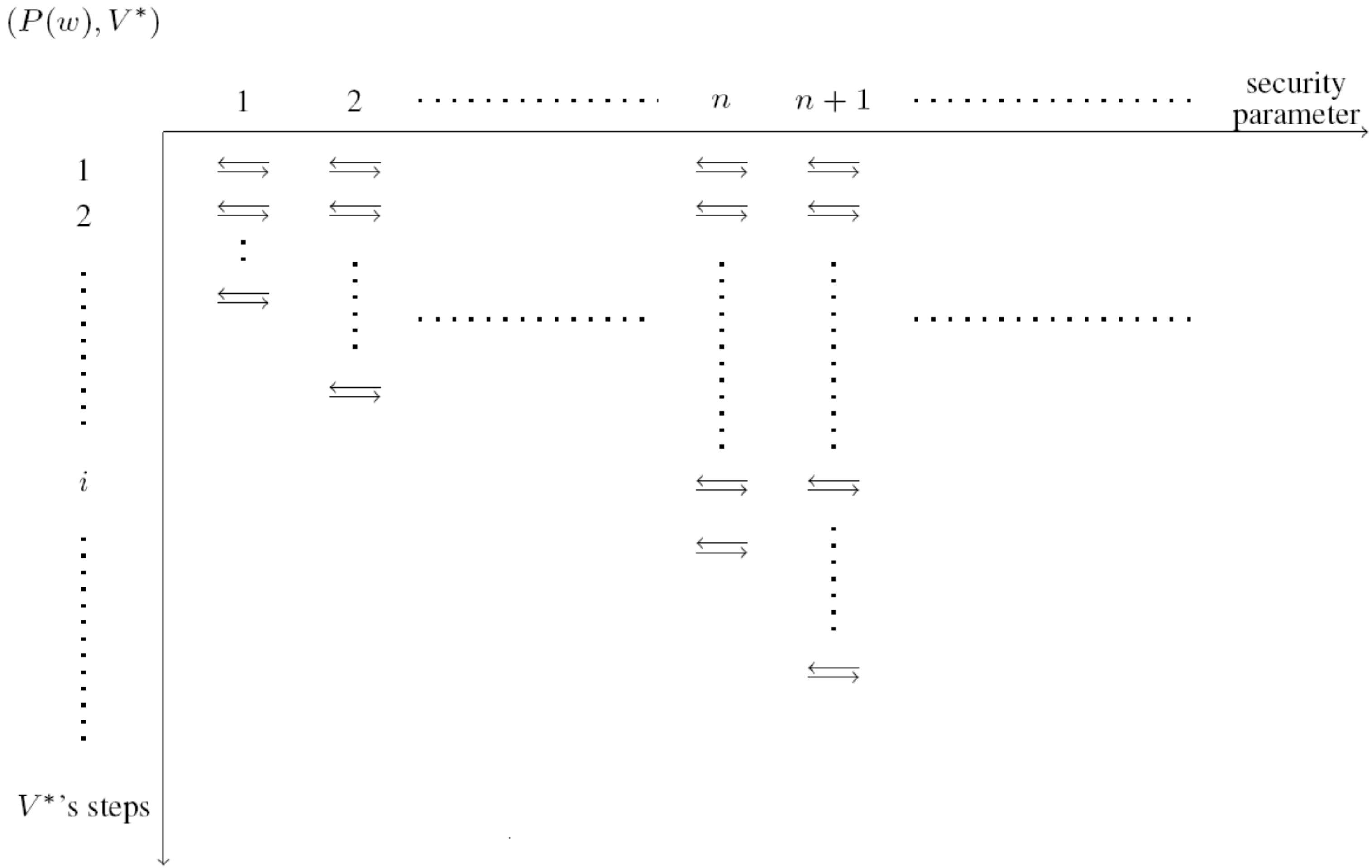
Proof of existence of an infinitely-many set $\{(n, i_n)\}$: A dissection of a magic V



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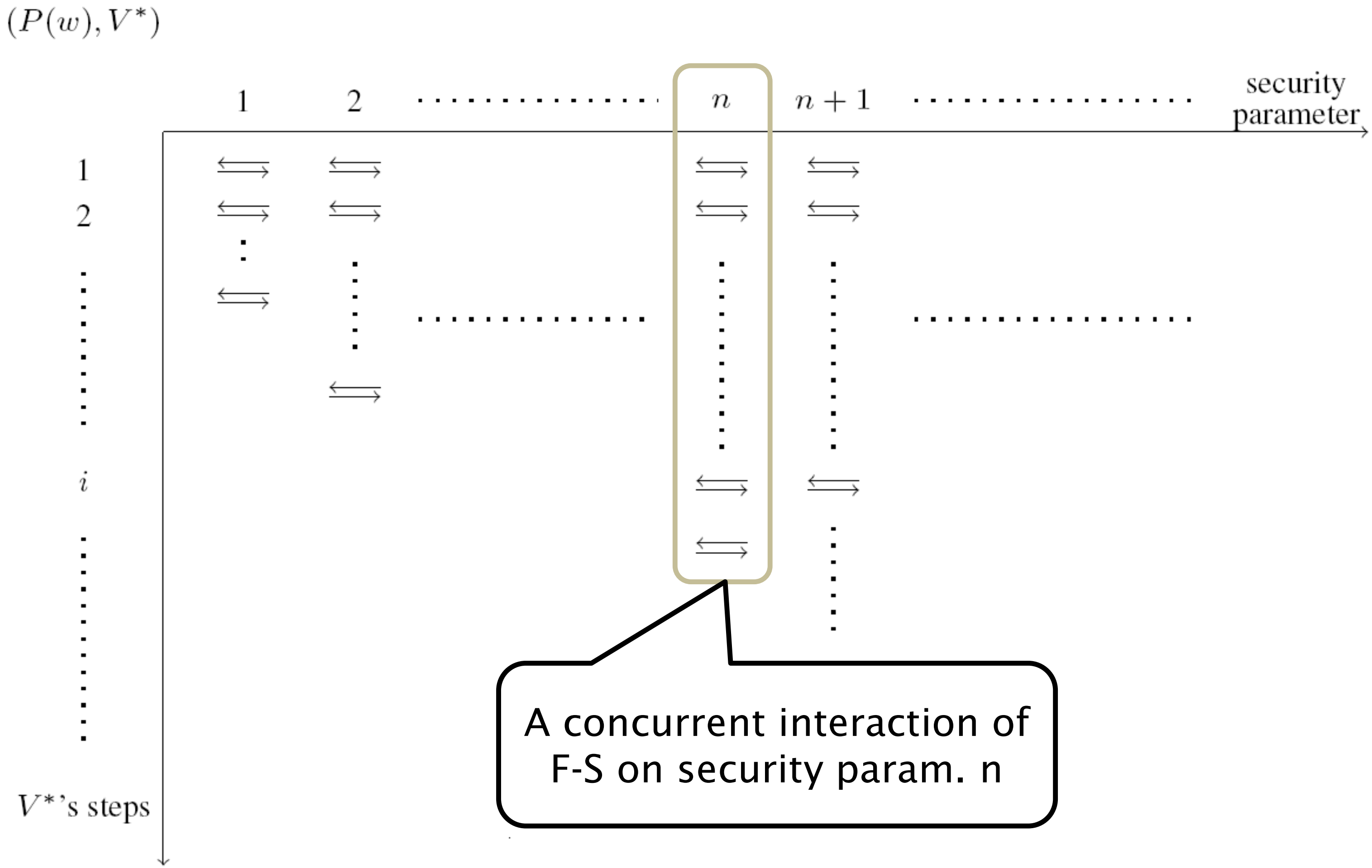

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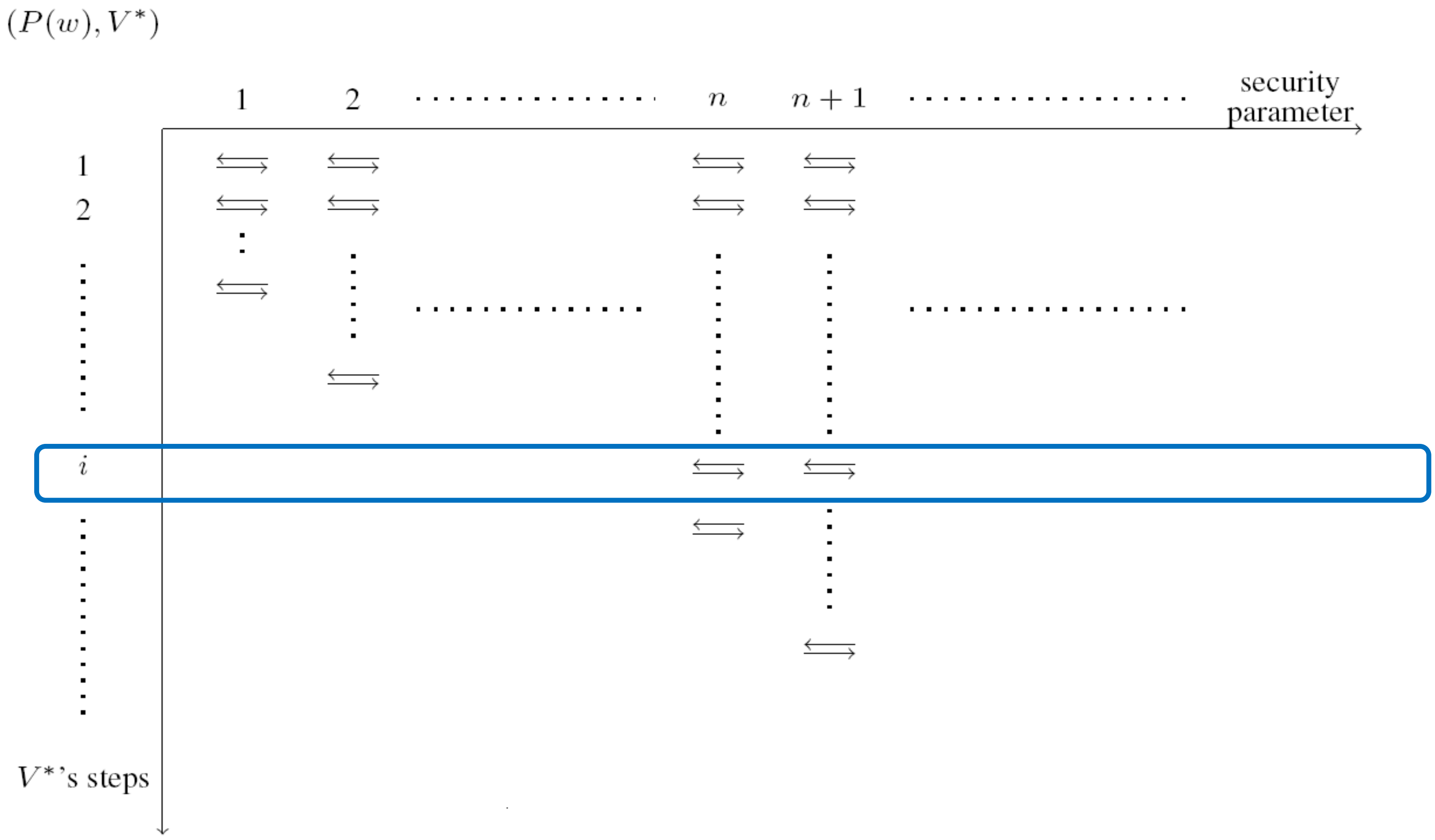

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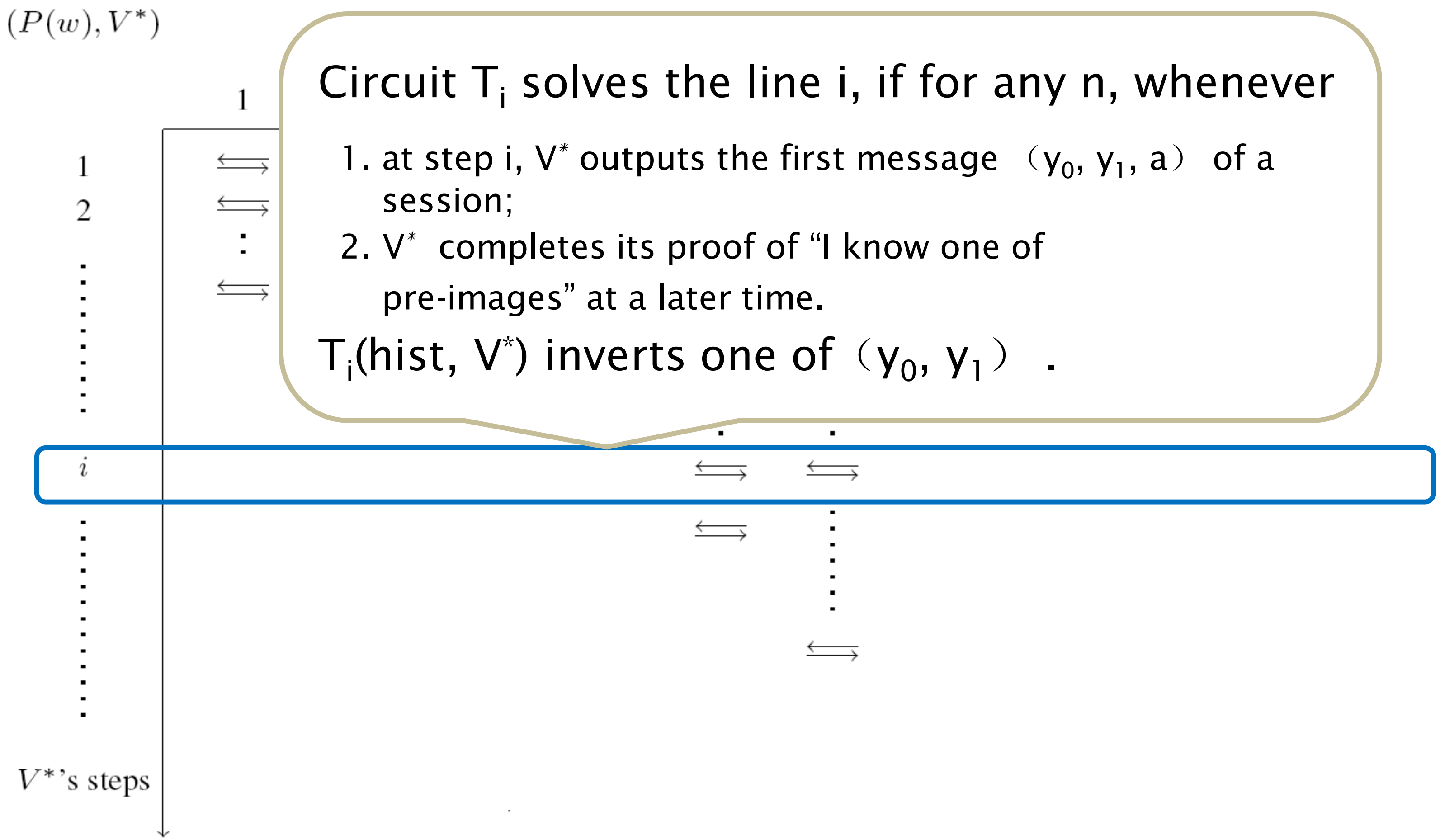

Circuit T_i solves the line i, if for any n, whenever

1. at step i, V^* outputs the first message (y_0, y_1, a) of a session;
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$T_i(\text{hist}, V^*)$ inverts one of (y_0, y_1).

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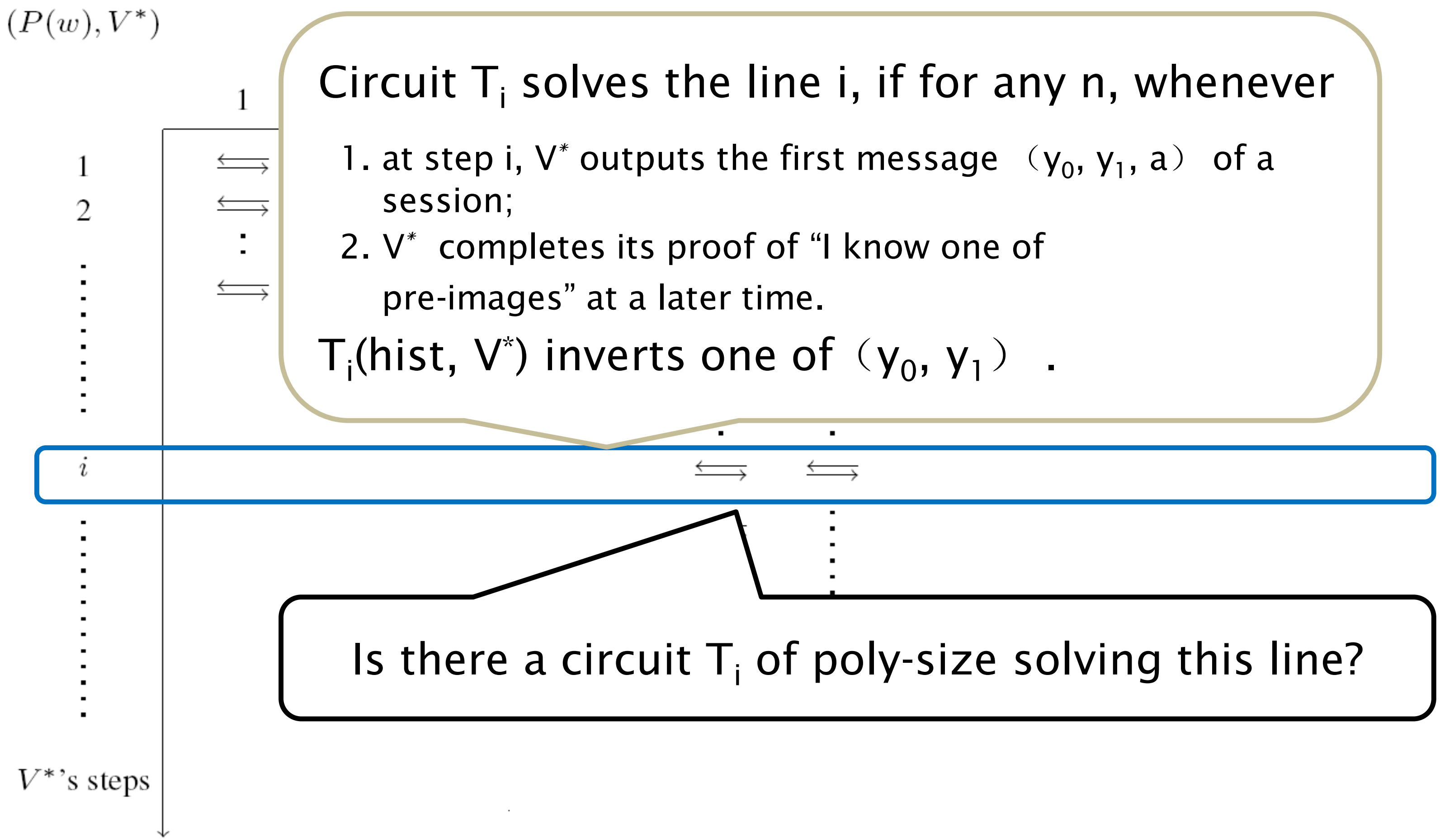

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Is there a circuit T_i of poly-size solving this line?

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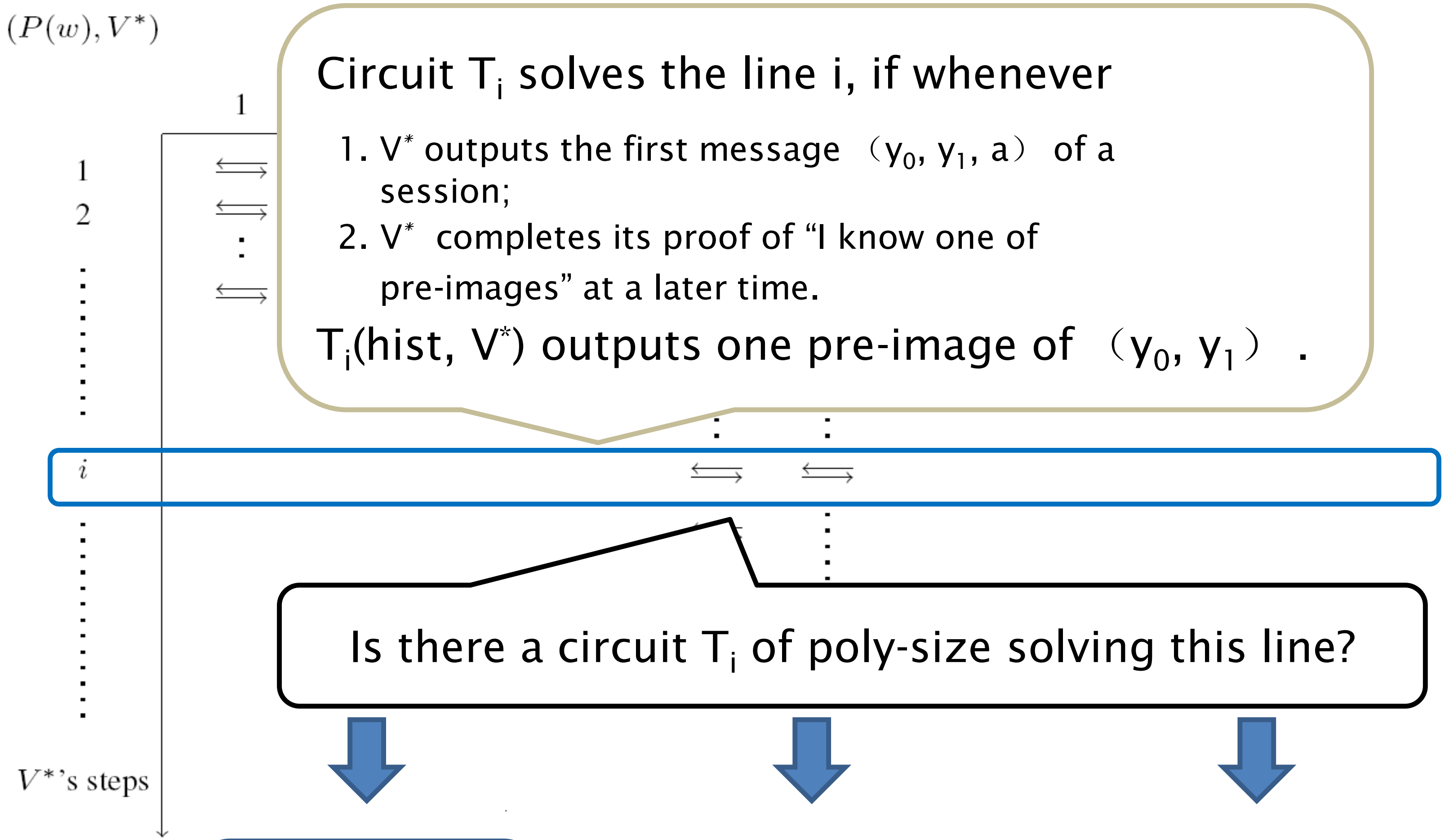

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We are done.

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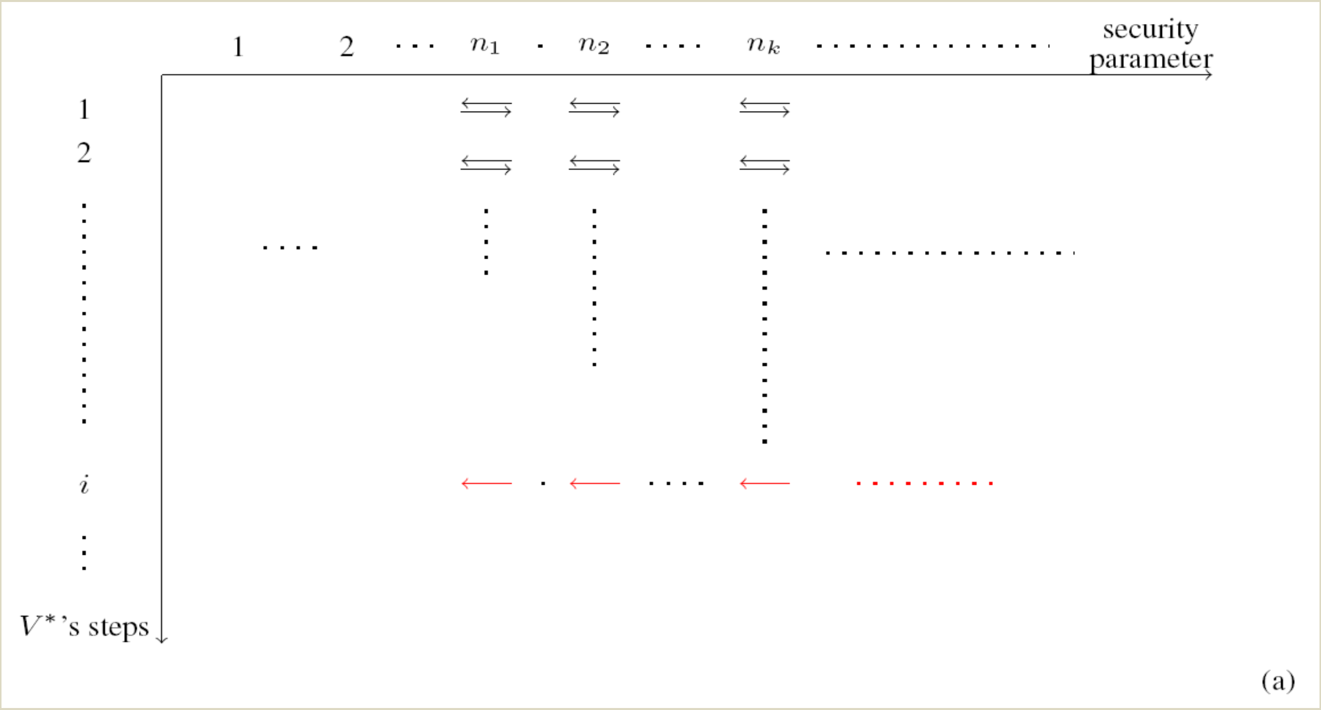

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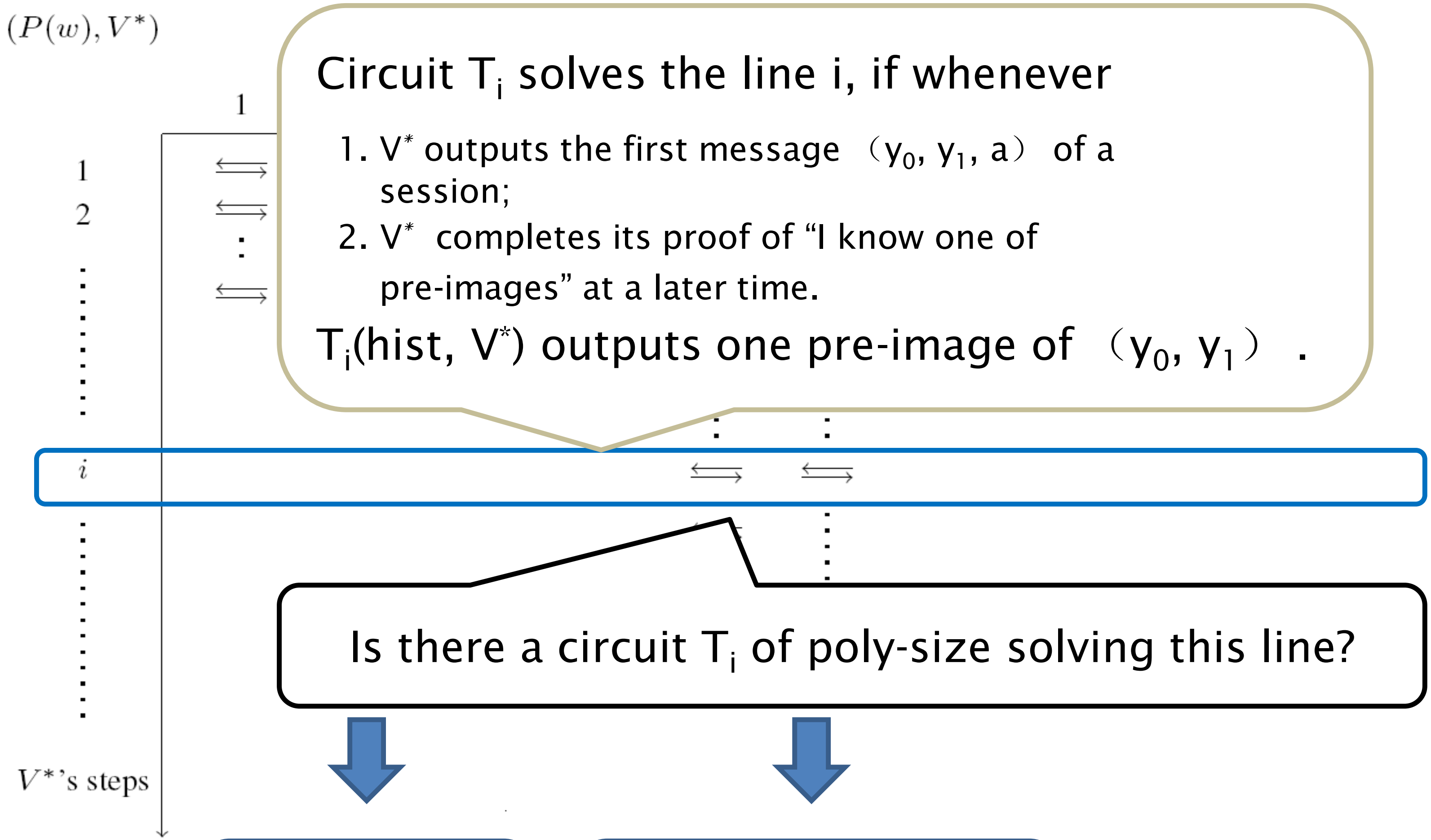

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YES，but requires size P_i，and no poly can upper-bound {P_i}.
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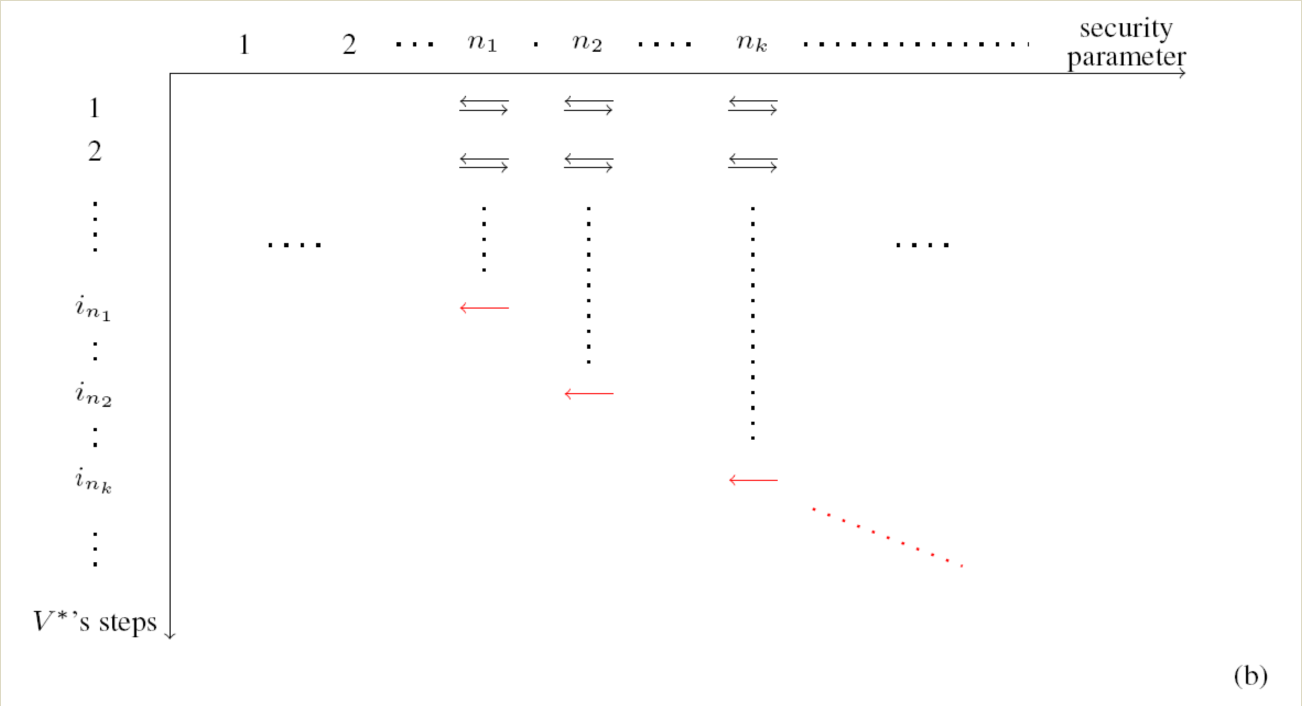

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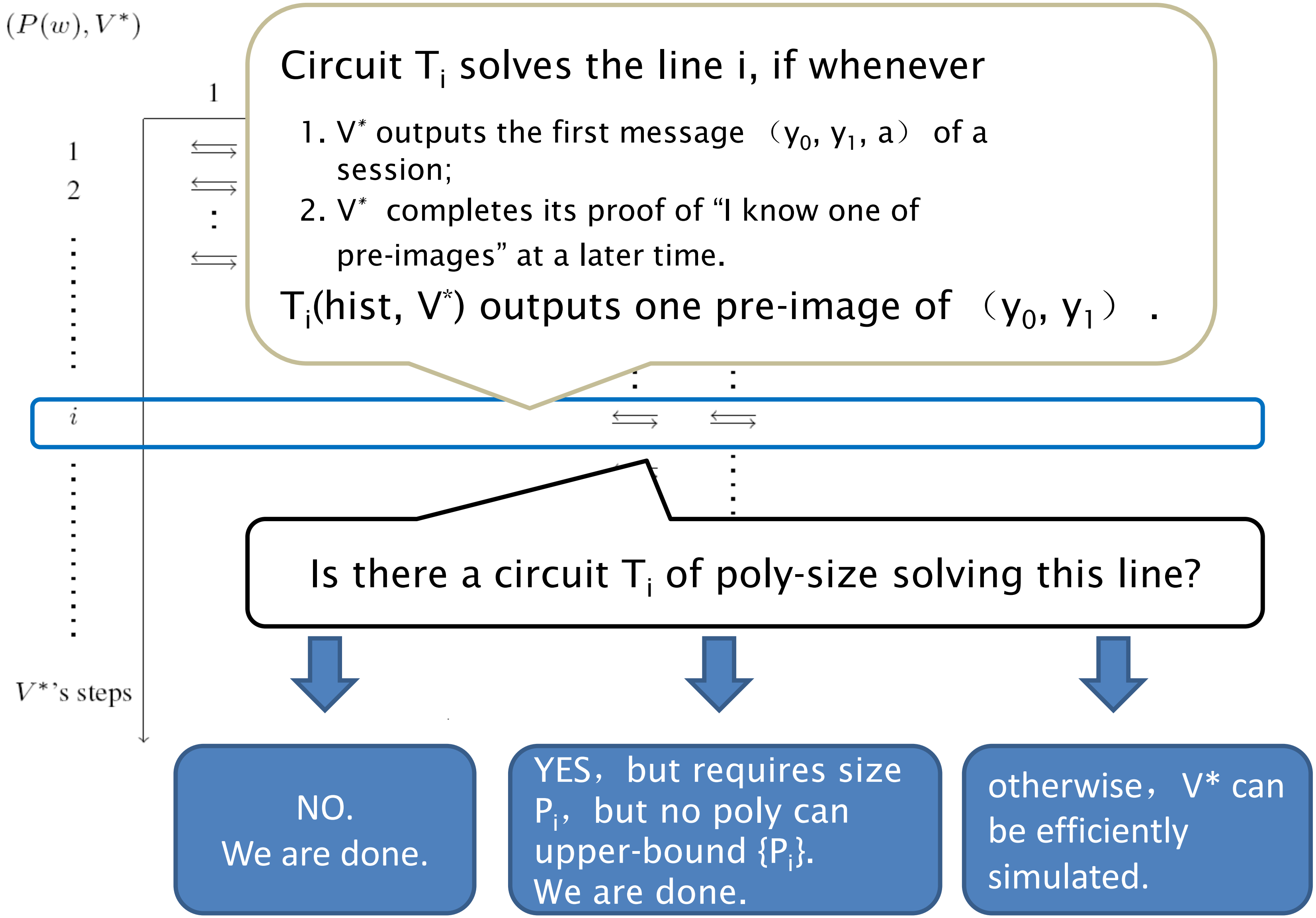
YES, but requires size P_i, and no poly can upper-bound $\{P_i\}$.
We are done.

Proof of existence of such an infinitely-many set $\{(n, i_n)\}$: A dissection of a magic V



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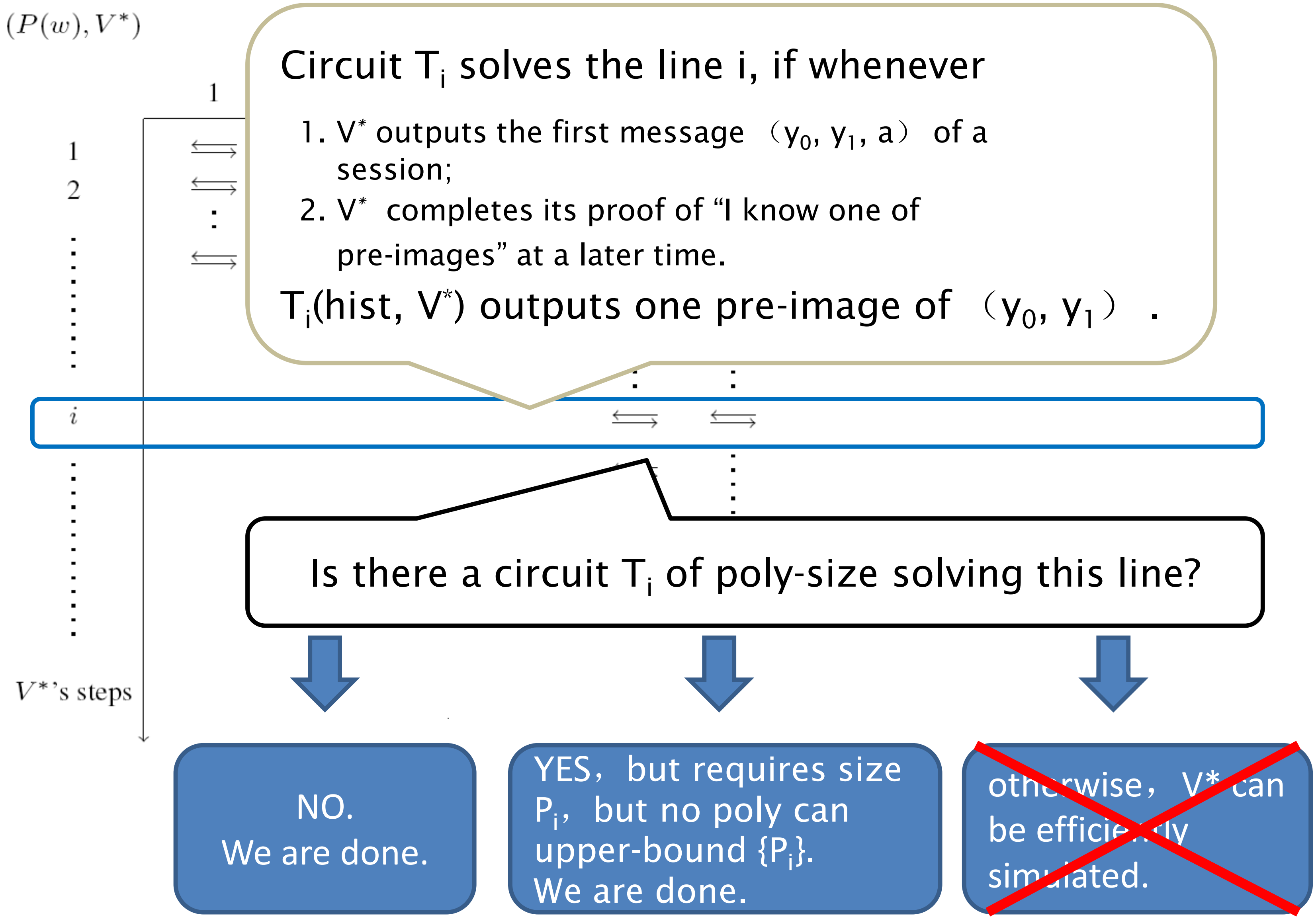


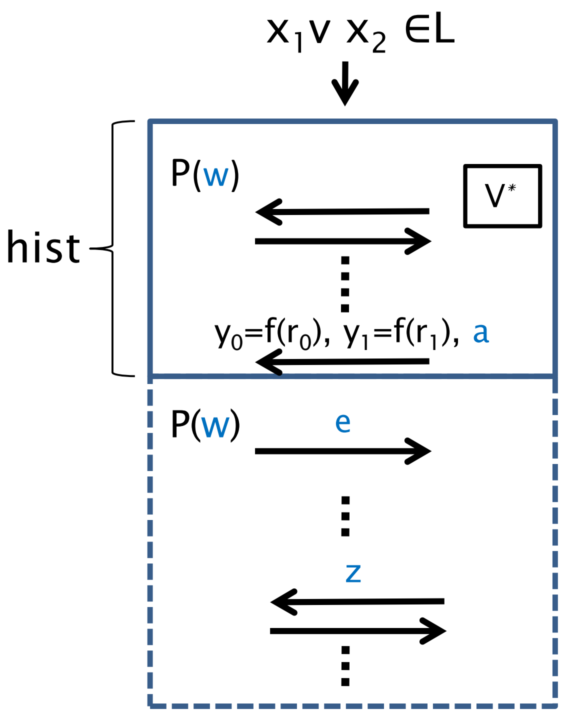
Now suppose that there is a magic V* that breaks the CZK of Feige-Shamir on OR NP-statements $(x_1 v\ x_2)$

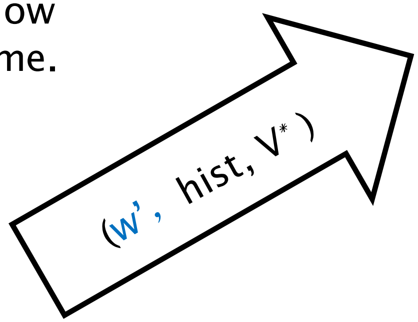
Consequence of a magic adv V^* on $x_1 v\ x_2$ *(oversimplified)*

There are infinitely many n, for each n

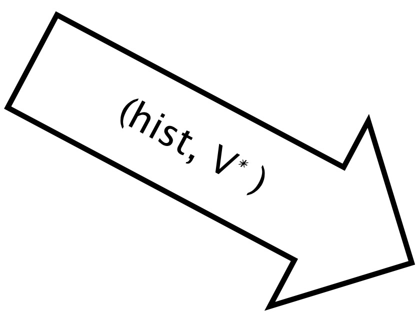
∃ V^*' step i_n

1. At setp i_n, V^* outputs the first message (y_0, y_1, a) of a session;
2. V^* completes its proof of "I know one of preimages" at a later time.





Given the witness w' as input, there is a PPT extractor E, E (w',hist, V^*) can extract r_b.
(by rewinding)



Without knowledge of any witness w, NO PPT T can extract r_b.
(otherwise we will have a simulator for V^*)

Consequence of a magic adv V^* on $x_1 \vee x_2$ *(oversimplified)*

There are infinitely many n, for each n

$\exists$ V^* ' step i_n

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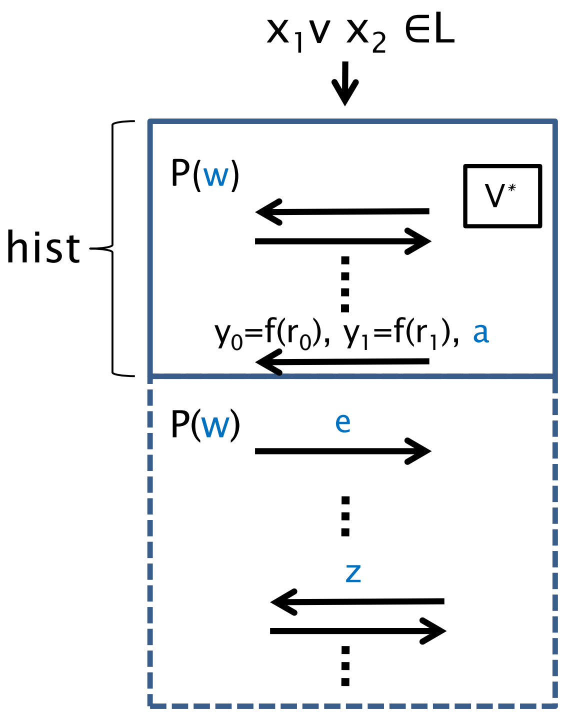

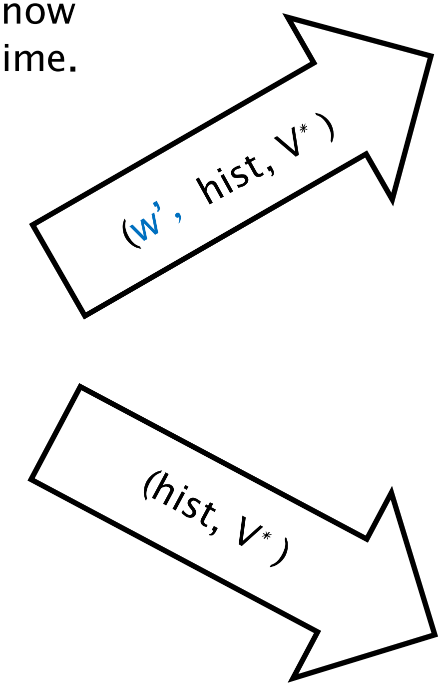

Given the witness w' as input, there is a PPT extractor E, E (w',hist, V^*) can extract r_b. (by rewinding)

Any valid witness to $x_1 \vee x_2$ will work due to concurrent WI of the Feige-Shamir.

Without knowledge of any witness w, NO PPT T can extract r_b.
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Consequence of a magic adv V^* on $x_1 v\ x_2$ *(oversimplified)*

There are infinitely many n, for each n

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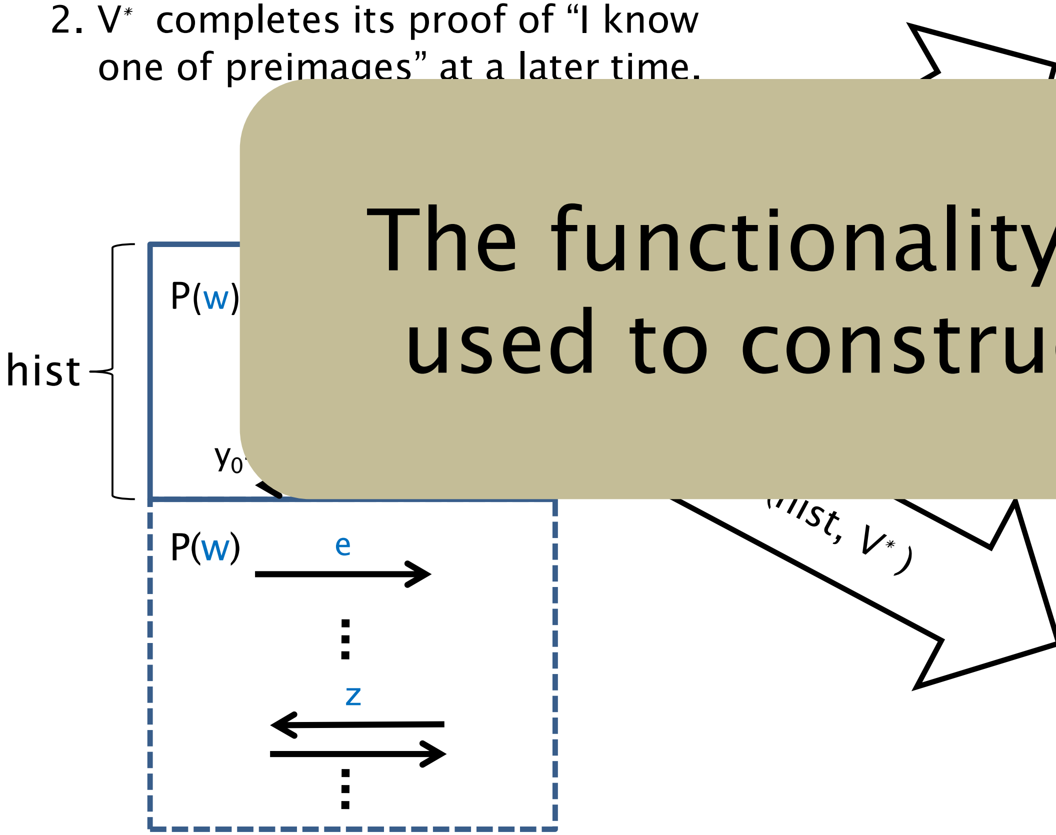
Given the witness w’ as input, there is a PPT extractor E, E (w’,hist, V^*) can extract r_b.

The functionality of V^* can be used to construct a PKE/KE.



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PKE/KE from Injective OWF（high-level idea）

Session key k ∈{0,1}

A

B(k)

PKE/KE from Injective OWF（high-level idea）

Session key $k \in \{0,1\}$

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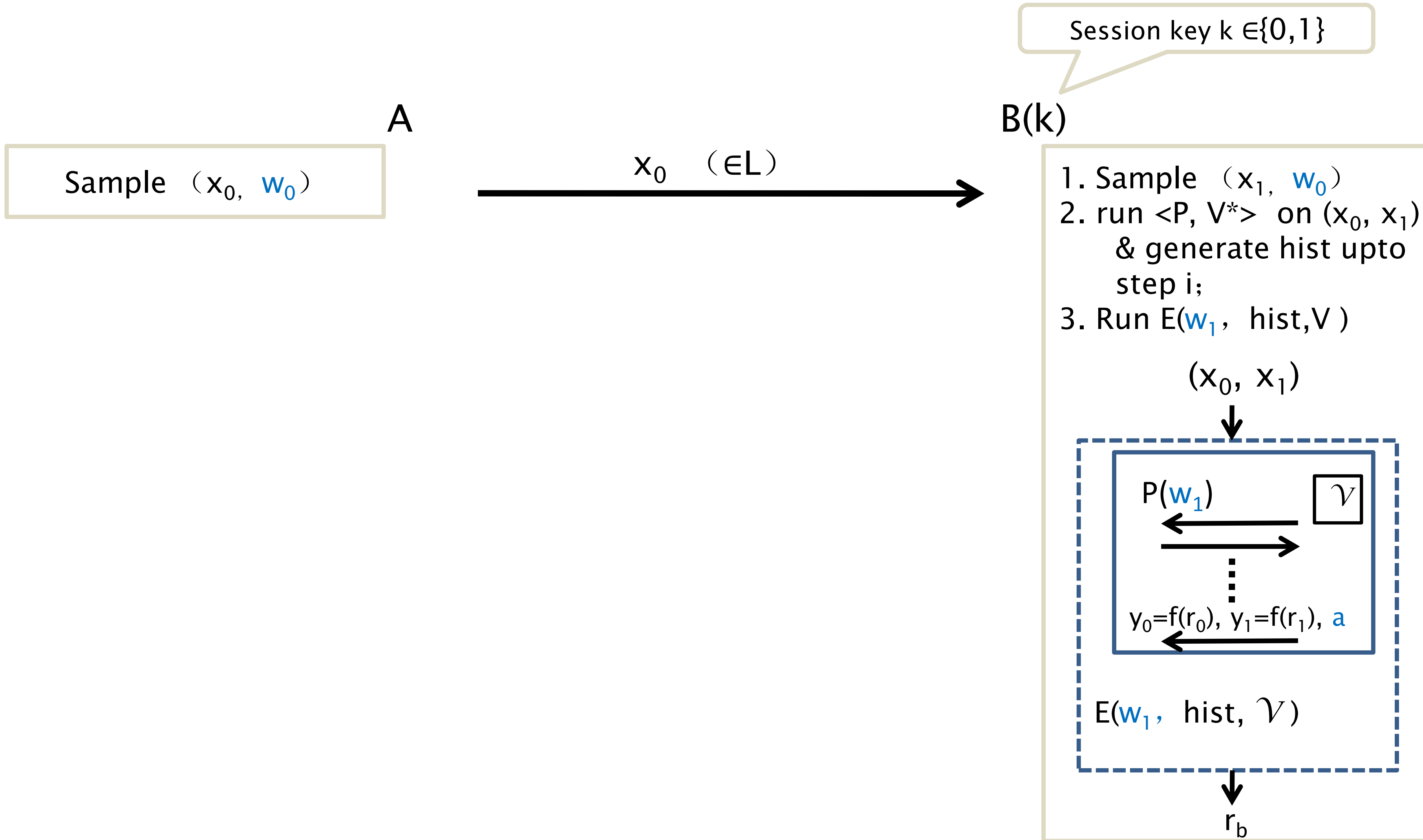
B(k)

Sample（x_0, w_0）

x_0 （$\in L$）→

PKE/KE from Injective OWF（high-level idea）



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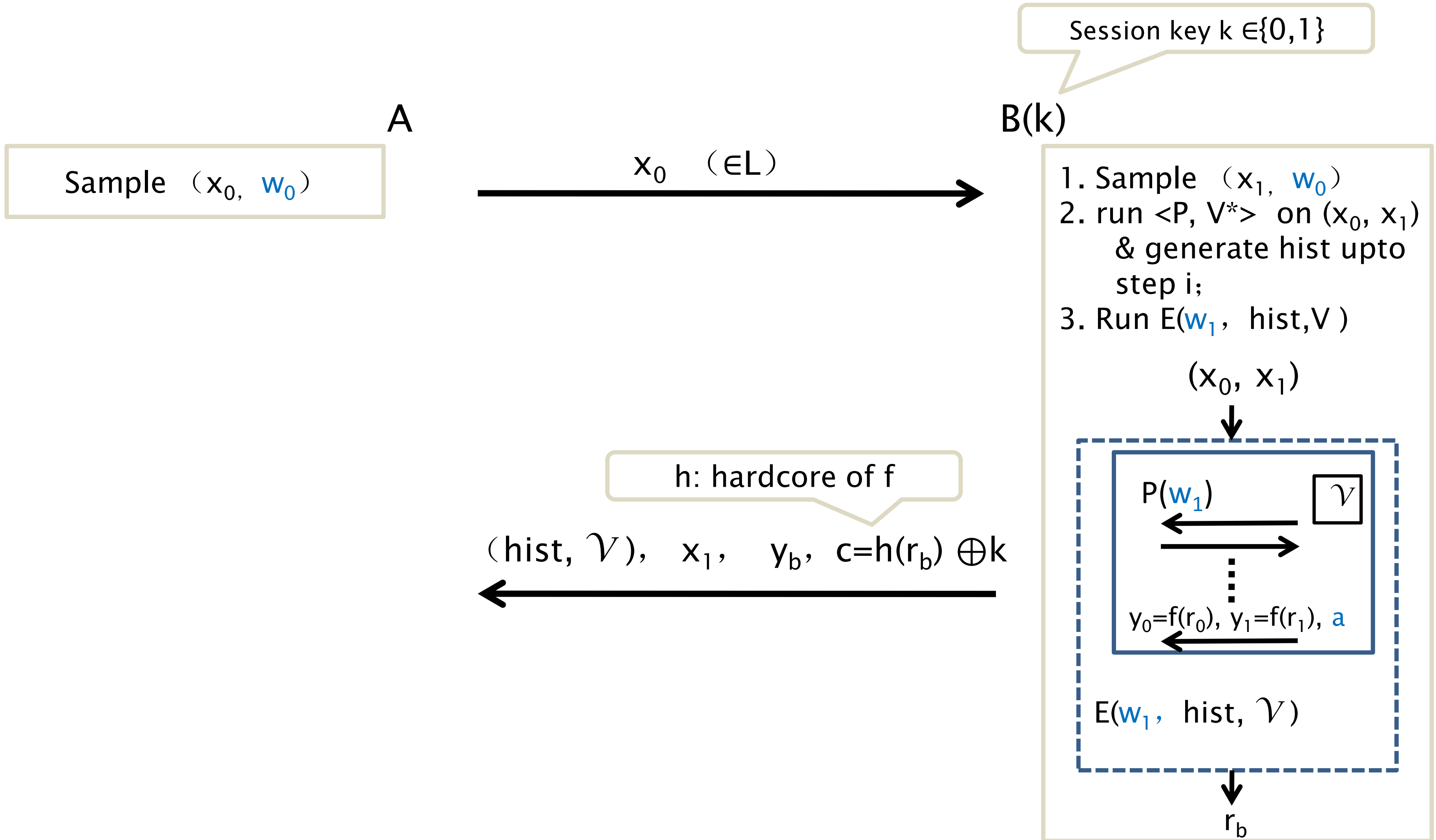

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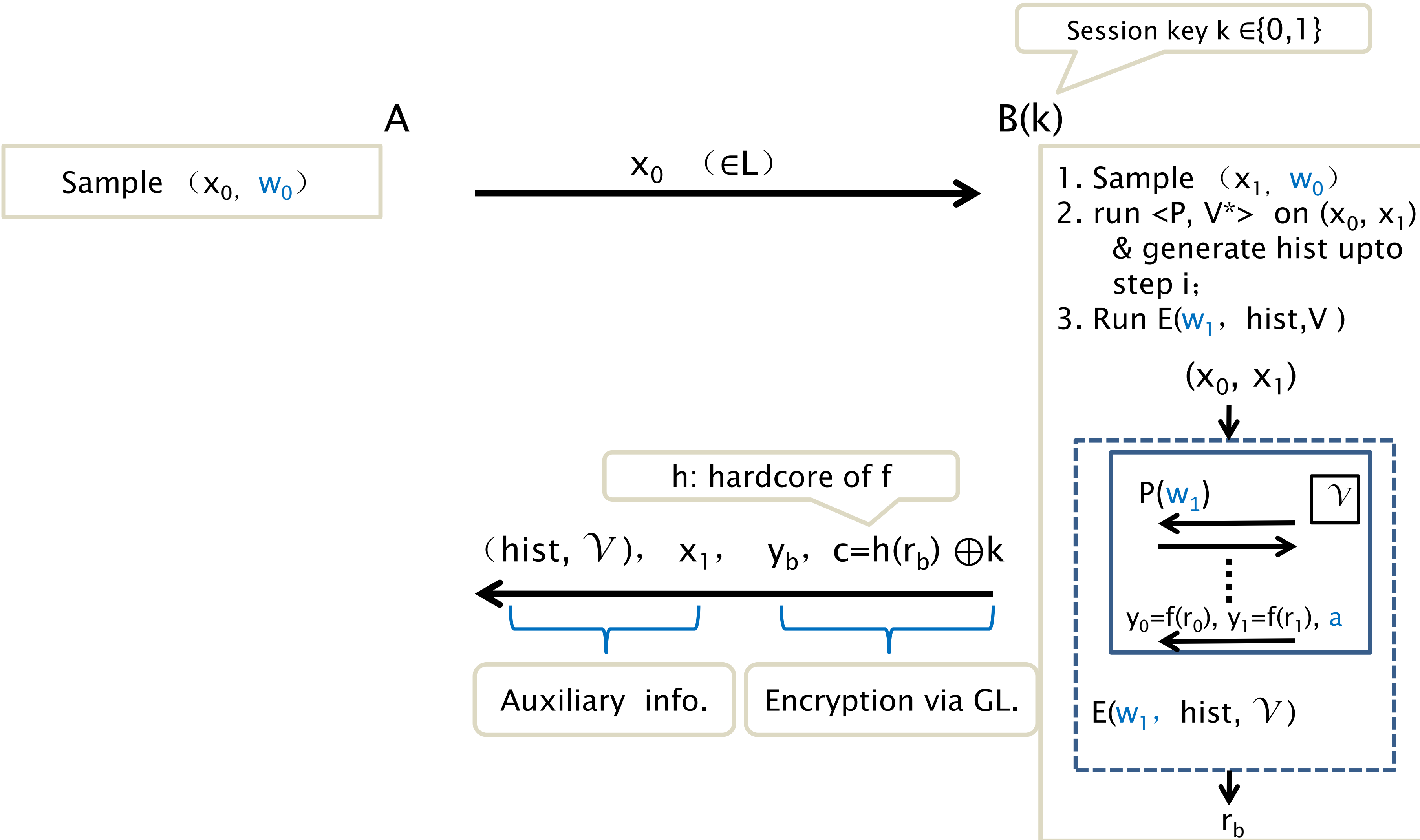

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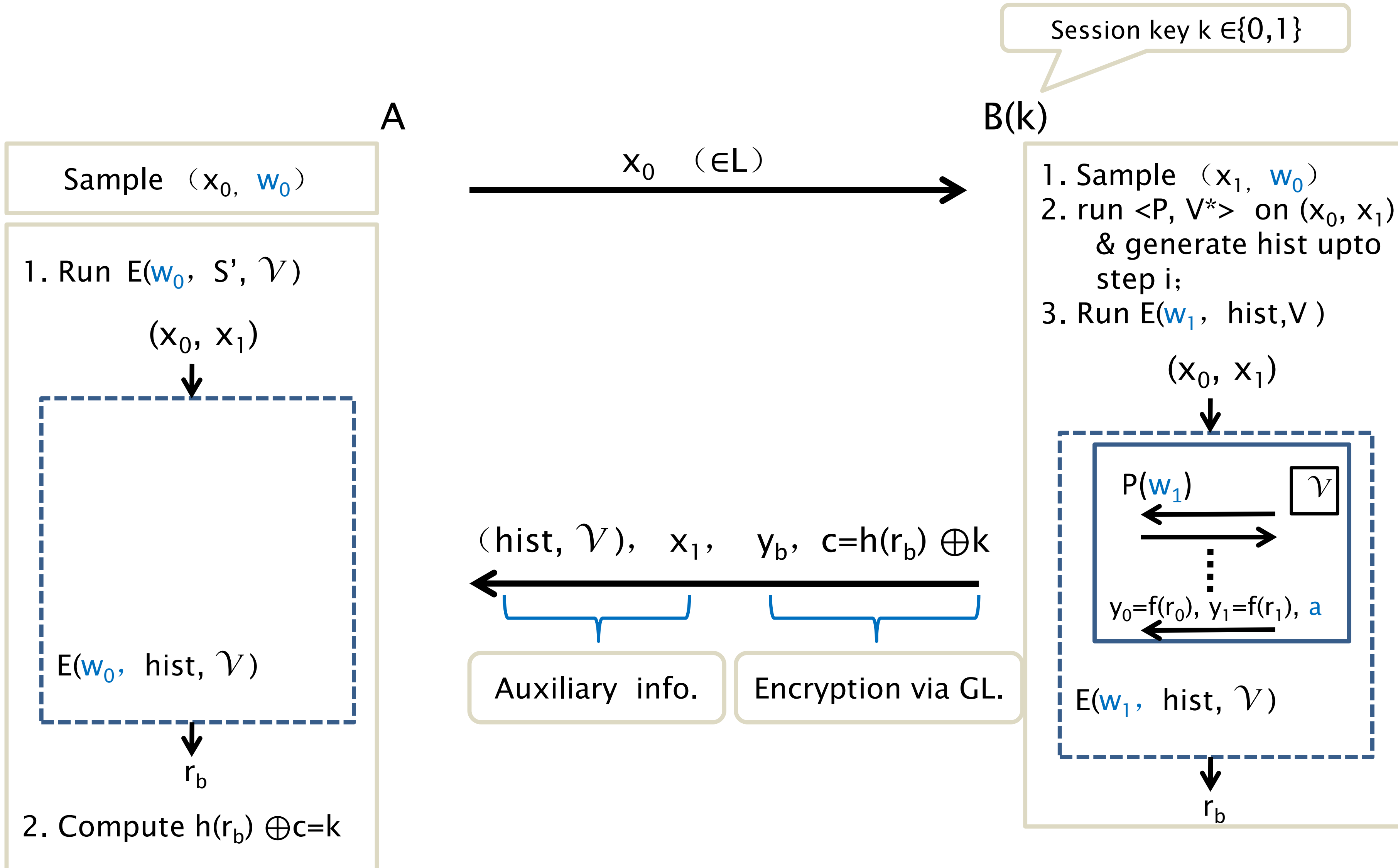

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- V* may output the first msg (a pair of images of f) at its step i (and complete the corresponding WI proof) *with some (non-negl) probability< 1*, which will introduce some error to our encryption and decryption algs.
  We use standard technique （parallel repetition）to reduce this kind of error.

Summary

We prove an ugly theorem:

Assume one-way function exists, then one of the following statements must be true:

1. (infinitely-often) PKE/KE exist.

2. The 4-round Feige-Shamir protocol is distributional concurrent ZK for OR NP-statements with small dist. gap.

$\forall$ V* $\exists$ S

$\exists$ S $\forall$ V*

Thank you!

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